The Difference between Pure and Applied Mathematics

Angela Simones

Brooks Preparatory

There is none; that's my conclusion. But of course, applied mathematics is mathematics done with some other end in view, whereas pure mathematics is an end in itself. Whether we call some mathematics "applied" depends on what the ultimate end is. For instance, one might say that the theory of solvable groups is "applied" because it can be used to develop Galois theory. This probably isn't what most people have in mind when they talk about applications of mathematics. So only some ultimate ends make mathematics "applied." As an aside, I want to mention that what Morris Kline, in his immensely influential essay[1], refers to as "applied mathematics" could be labelled more accurately as pure mathematics whose subject is nature. He argues that such is the only ultimately worthwhile direction for research, but the question of "utility" is not important for him, so the point I'm making is only tangentially related to his work.

We could say that mathematics is applied if its point is to meet basic human needs such as food, shelter, and warmth. Of course there are derivative needs, which would make some mathematics applied even if it doesn't directly help to supply food, shelter or warmth. For instance, one could argue that military strength is necessary to insure basic human needs, so that mathematics which increased military strength would be applied. To define applied mathematics from here it is only necessary to iterate. Applied mathematics is mathematics that solves problems which (possibly in a highly derivative way) meet basic human needs. For instance, theorems on the plane crossing numbers of graphs have applications to VLSI design, which leads to new computer architectures. Computers expedite food production, and so calculating plane crossing numbers of graphs is applied mathematics.

We can make the same sort of distinction in the field of metal working: Making spoons, thimbles, or counterfeit coins is applied metal work; spoons help people eat and thimbles help them keep warm (in a derivative way). On the other hand, making statues of horses out of bronze is pure metal work. Now we can see the first problem with the pure/applied distinction. People don't just make spoons, they make pretty spoons. In fact, people who make spoons spend a lot of time thinking about making their spoons prettier. This instinct has led spoon designers to make spoons which aren't even meant to be used - they're meant to be displayed. Some people put them in frames and hang them on their walls, so is spoon manufacture pure or applied? The same thing happens with thimbles, and the same thing happens with mathematics.

For instance, as mentioned above, computer scientists study crossing numbers of graphs for the purpose of improving VLSI techniques. Recent work in this area is surveyed in[2], whose authors are all computer scientists, employed in computer science departments, and publishing papers in computer science journals. And yet, they've put a lot of thought into the crossing numbers of graphs on nonorientable surfaces, a study which they certainly aren't pursuing for the sake of improving chip design. My claim is that even mathematics which is begun with the purpose of meeting basic needs will end up as mathematics done for its own sake. The natural end of applied mathematics is pure mathematics, because people naturally want to make even their utilitarian creations beautiful, and the beauty ends up becoming the purpose. The reason for this is that people have a basic need for beauty, truth, spirituality, and a productive life as well as for food, warmth, and shelter. "Man does not live by bread alone, but by every word that proceedeth out of the mouth of God."[3] or "Hearts can starve as well as bodies - give us bread, but give us roses."[4]

Now I want to turn the above line of reasoning upside down. It isn't really true that people begin their applied mathematical activities by trying to meet bodily needs, and subsequently develop them to meet spiritual needs. Actually bodily needs and spiritual needs are inextricably linked, and neither is prior to the other. In primitive societies, where people make their living by hunting and gathering, these are not merely utilitarian pursuits, but spiritual activities-expressions of humanity's place in the universe. If human needs in the large sense are being met, this will always be the case. People won't work effectively merely to feed themselves - If people are left to have their own way, work will always be at the same time an expression of human spirituality. Also, it probably isn't possible to subsist bodily if one's only goal is to get food, warmth, and shelter.

Just as a tennis player has to follow through in order to hit the ball effectively, people have to embed their pursuit of bodily requirements in an infinitely richer context in order to be able to meet them at all. Without the larger spiritual context, there's really no point in meeting bodily needs, and without meeting the bodily needs, there's no possibility of the larger spiritual context, so it doesn't make sense to say that one is prior to the other. So in the end there is no difference between pure mathematics and applied mathematics. Both are activities pursued for their own sake, or rather for the sake of living a fully human life.

REFERENCES

1. Kline, Morris. Mathematics: The Loss of Certainty. Ox- ford University Press, Oxford. 1980.
2. Shahrokhi, F., Sykora, 0., Szèkely, L.A., Vrt'o, I. "The Crossing Number of a Graph on a Compact 2-mani- fold." Adv. Math. 123(1996), 105-119.
3. Matt. 4:4
4. "Bread and Roses." Words by James Oppenheimer, music by Caroline Kohsleet. In: Little Red Songbook (36e). International Workers of the World Press. Ypsilanti, 1997.