Permutations and Combinations

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ABSTRACT

Mathematics from past times has proved to be one of the difficult subject probably because it requires too much of reasoning and greater thinking capacities. As if that is not enough, the poor methods of presentation and unfriendly attitude of some teachers makes the situation to appear difficult such that mathematics continues to attract the interest of very few people and the percentage of females among them is very insignificant. Most of the students studying mathematics resulted to it because they had no otherwise by not being admitted to the course of their choice. Permutation and combination is one of the topics in Mathematics that pose problems to students. In this paper, we explain how permutation and combination could be taught using play way approach among other methods that could be used. At the same time in this paper a new method of generating permutations in a more orderly manner is developed.

FOCUSING QUESTIONS

In a given question, how do we recognize that it asks whether for permutation or for combination?

And where does order matter between permutation and combination?

INTRODUCTION

Combinatorics is the branch of mathematics which deals with collections of objects that satisfy specified criteria like counting arrangements, permutations, and combinations. For example, combinatorics would answer the question "how many different ways can you arrange a 20-song playlist if you have 60 songs to choose from?"

Combinations is the branch of combinatorics where changing the order of the objects does not have a greater meaning .For example, the question "how many teams of 10 baseball players can a manager assemble from a roster of 18 players?" is a combinations question since changing the order in which the player is chosen does not create a new arrangement. Permutations is the branch of combinatorics where changing the order of the objects creates a greater meaning. For example, the question "how many different ways can a baseball pitcher who throws 6 unique pitches throw the next 3 pitches?" is a permutations question because changing the throwing order creates a new arrangement like, throwing fastball, sinker, changeup is different than throwing sinker, changeup, fastball.

BODY

Permutations involve problems in where the arrangement of items is taken into. Some examples of common problems are arranging books on a shelf, creating a seating chart or making a schedule and time table. In all of these situations, moving things around creates a different arrangement, chart, or schedule. Suggesting, if changing the order of the items in question creates a uniquely different arrangement, then one is dealing with a permutation.

There are two general ways to solve these problems. The first method involves drawing the problem. The second involves the use of a formula. For simpler problems it is often possible to draw the arrangement. However, as the problems become more difficult, drawing the problem can become more prone to errors and using the formula may be best. In permutation we have basically two types of permutation, which are permutation with repetition and permutation without repetition.

Permutation with repetition

When we have a total number of items to be n, this simply means that we have n choices each and every time. On choosing r of them, we have the permutation to be $n \times n \times ...$ (r times).this can be translated that for the first choice we have n possibilities, then for the second choice there are n possibilities and with regard to that it keeps on multiplying each time. It now becomes cheaper to write down as an exponent of r. $n \times n \times ...$ (r times) = nr, where n is the number of items to choose from and r are chosen from them.

Permutation without repetition

Here we are required to make a reduction of the number of available choices each time. It should also be noted that without repetition our choices get reduced each time and it is mathematically written as a factorial function (!) that means to multiply a series of descending natural numbers. The formula is written as $\frac{n!}{(n-r)!}$ where n is the number of things to choose from and we choose r of them without repetition and order matters.

Combinations

Permutations are not similar to combinations since combinations do not consider order. The number of possible euchre hands or the number of teams that can be created in a gym class are just a few examples. In both cases, the order does not matter. Unlike permutations, combinations cannot be easily solved by diagramming the problem. They are also of two types consisting of combinations with repetition and combinations without repetition. Majorly under combinations without repetition, the numbers are drawn one at a time. This means that the order does matter and then it alters so that order does not matter. Hence we adjust the permutation formula to reduce it by the number of ways that the objects could be ordered. This is due to the reason that we are not totally interested in their order.

$$\frac{n!}{(n-r)!} \times \frac{1}{r!} = \frac{n!}{r!(n-r)!}$$

Hence,

$$\frac{n!}{r!(n-r)!} = \binom{n}{r}$$

where n is the number of items to select from and r are chosen from them.

DISCUSSION

Permutation without Repetition sample question

How many different arrangements is there for the letters a, b, c, d, e?

This is a permutation without repetition because an arrangement of a, b, c, d, e must include all the letters a, b, c, d, and e. For, example, a, a, b, b, b is not an arrangement of a, b, c, d, e. Therefore, this is a permutation without repetition.

If you choose r objects out of a group of n distinctobjects, You get NPR= n! / (n-r)!

For this question, we are choosing 5 objects (a, b, c, d and e) out of 5 objects (a, b, c, d, e), so we get:

5P5= 5! /0! = 5*4*3*2*1 / 1 = 120 (Remember: 0! = 1)

Permutation with repetition sample question

If you look at the word TOOTH, there are 2 O's in the word. Both O's are identical, and it does not matter in which order we write these 2 O's, since they are the same. In other words, if we exchange 'O' for 'O', we still spell TOOTH. The same is true for the T's, since there are 2 T's in the word TOOTH as well. In how many ways can we arrange the letters in the word TOOTH?

We must account for the fact that these 2 O's are identical and that the 2 T's are identical.

We do this using the formula:

nPrx1! X2! Where x is the number of times a letter is repeated.

nPrx1!x2!5P52!2!5P52!2!5P52!2!=5P52!2!=5×4×3×2×12×1×2×1=1204=30

We can arrange the letters in the word TOOTH in 30 different orders

Combination

In how many ways can a group of five players on the varsity boys' basketball team be chosen from a team of 14 players?

This represents a combination of 14 objects taken 5 at a time and is equal to 2,002.

14C5=14/ (9! 5!) = (14*13*12*11*10)9! =2002

In conclusion can clearly see from the examples above that if n become bigger, the task of exhaustively listing and counting all the possible outcomes of the arrangements becomes a difficult task. Hence we cannot effectively carry out this without the use of computer. To determine the number of possible outcomes we need an elegant mathematical formula like the results we have obtained above.

REFERENCES

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