Study on PID Gain of Active Suspension Using QuarterCar Model and Simulink

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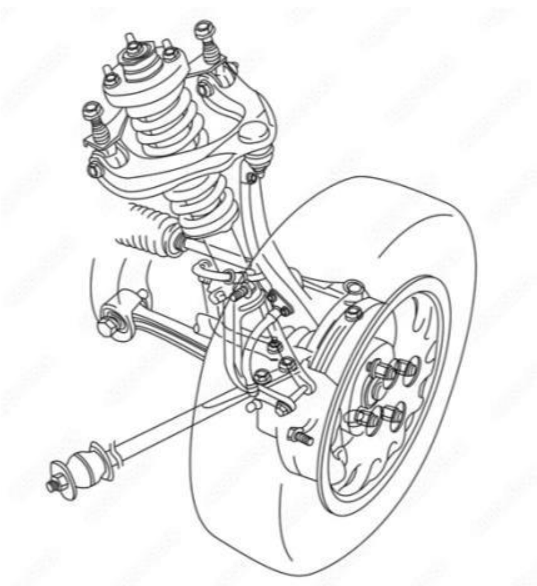
Abstract

The goal of this research is to use the quarter car model and explore ways in which active and passive suspension can be applied for vehicle control. First, different state-space representations will be created using bond graphs. Once the analysis of the plant is complete, the results will be modeled using Simulink. To improve the performance of the original plant, the gain values of the control system will be adjusted. After choosing the base value using the Ziegler-Nichols method, the gain value will be further tuned through trial and error. Based on the road conditions, the suitable K_p, K_i, and $K_d$values will be determined to design the control system.

1. Introduction

1.1 Motive



The suspension system is responsible for the stable connection between the main body of the vehicle. The stability from the suspension system enhances the ride quality and safety of the driver. A suspension without any control unit is comprised of a spring and a damper. By fine tuning the characteristics of the spring and damper, a passive suspension system can be created. However, due to the fixed characteristics of the spring and damper, a passive suspension system is not practical for real-life scenarios, as road conditions constantly change. Hence, using feedback control to create a dynamically adjust the characteristics will help create an optimal suspension system. [1]

1.2 Background Information

1.2.1 Laplace Transform

The Laplace Transform is a method to solve differential equations, as it simplifies the original function into a different "space" and helps solve this converted function. For example, a function that may be difficult to solve in A Space can be simplified in B Space and solved using simple arithmetic operations. Here, $A \rightarrow B$ or $B \rightarrow A$ is known as the Laplace Transform. [2]

$$F(s) = f\{f\}(s) = \int_{0^-}^{\infty} e^{-st} f(t) dt$$

The definition of Laplace Transform is stated above, where the Laplace transform of $f(t)$ is denoted as $f\{f\}(s)$.Using this definition, there are several properties:

Linearity:

$$f\{af(t) + bg(t)\} = af\{f(t)\} + bf\{g(t)\}$$

Differential:

$$f\{f'\} = sf\{f\} - f(0)$$

Integral:

$$f\{\int_0^t f(r)dr\} = f\{u(t) * f(t)\} = \frac{1}{s}F(s)$$

Convolution:

$$f\{f * g\} = f\{f\} \cdot f\{g\}$$

1.2.2 Bond Graph

Bond graphs, which is expressed through flow and effort, can be used to find differential equations that controlthe plant.

IRC Component:

The "I" component stores energy. It represents a machine system's inertia or an electric system's inductor.

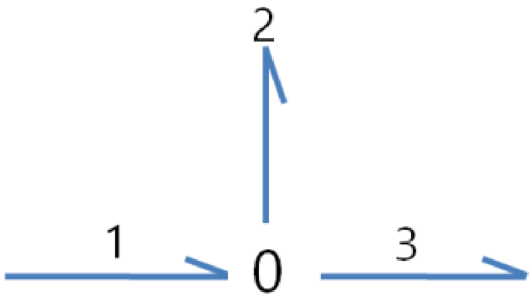
The "R" component uses the energy. It represents a machine system's damper or an electric system's resistor.

0 Junction:

A junction in a bond graph where all efforts are equal.

$\sum fin = \sum fout$

The "C" component stores potential energy. It represents a machine system's spring or an

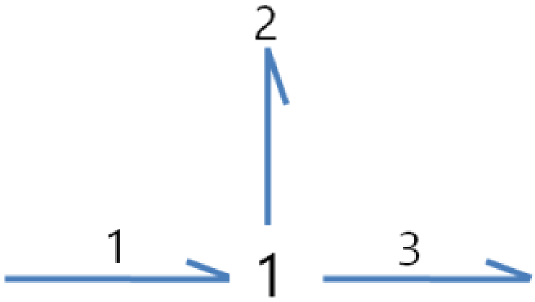


electric system'scapacitor.

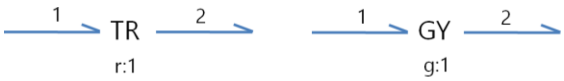
1 Junction:

A junction in a bond graph where all flows are equal.

$\sum ein = \sum eout$



Transformer/Gyrator:

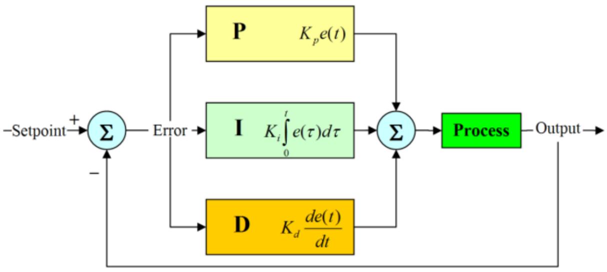


A transformer creates a connection between flow in and flow out, as well as effort in and effort out. It iscomparable to a gear, since it alters the ratio of the inputted component.A gyrator creates a connection between flow in and effort out, as well as effort in and flow out. An example is a DC motor, as it converts voltage to acceleration.

Causality

Causality can be used to identify the system's variables and whether the power variable is dependent or independent. Causal analysis can also help determine state-space equations. [3]

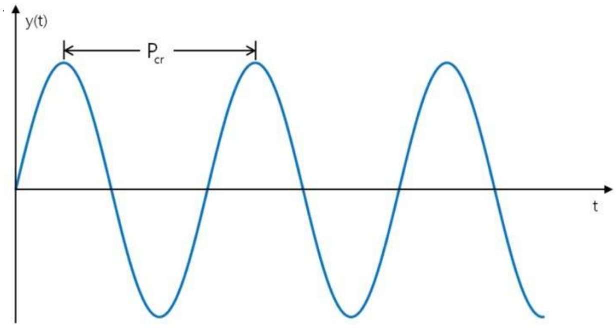
1.2.3 PID Control



PID control is a type of feedback control. During this process, the output of the function that will be controlled is compared with the target value and the error value is calculated. Using this error value, the correct manipulated variable is calculated and outputted. To find the correct manipulated variable, a proportional value of the error function, as well as its derivative and integral is used. [4]

1.2.4 Ziegler-Nichols Method

The key aspect of PID control is determining how to change the error function using the gain value. One method to set the gain value is the Ziegler-Nichols Method. Within a given plant, the Integral gain value of the PID gain value is set to infinity, the Derivative gain value is set to its minimum, and only the proportional gain value is increased, which will help find the oscillating ultimate gain (K_{cr}). Using the ultimate gain value, the ultimate period (P_{cr}) can be determined as shown below. [5]



Using the two calculated values, K_{cr} and P_{cr}, the gain values of different controllers can be found through using the table and equations shown below.

Type of Controller	K_p	T_i	T_d
P	0.5 K_{cr}	∞	0
PI	0.45 K_{cr}	$\frac{1}{1.2} P_{cr}$	0
PID	0.6 K_{cr}	0.5 P_{cr}	0.125 P_{cr}

$$u(t) = K_p e(t) + K_i \int_0^t e(r)dr + K_d \frac{de(t)}{dt}$$

$$C(s) = K_p(1 + \frac{1}{T_i s} + T_d s)$$

The "S" in the equations above is Laplace Transform, so the middle term is integrated and the end term isdifferentiated. Using the equations and table above, the following equations can be created. [6]

$$K_p = 0.6 \cdot K_{cr},\ K_i = 1.2 \cdot \frac{K_{cr}}{P_{cr}},\ K_d = \frac{3}{40} \cdot K_{cr} \cdot P_{cr}.$$

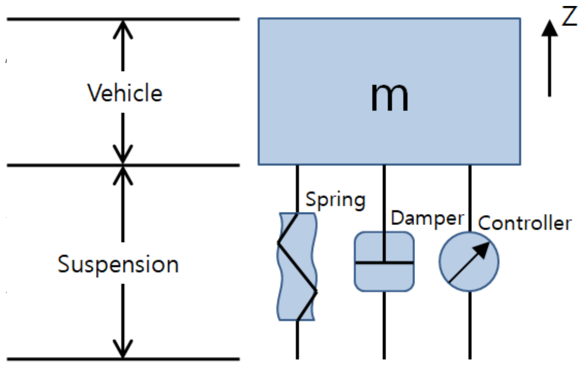
However, the gain values that are found using this method must be further tuned based on the plant.

2. Body

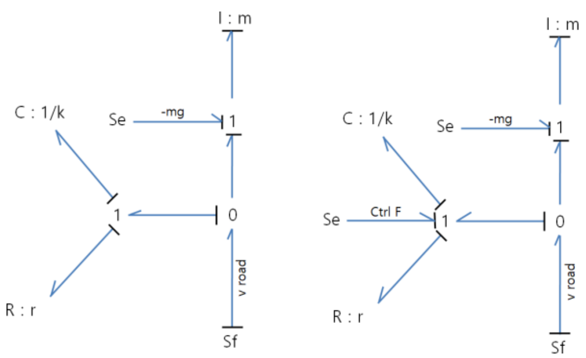
Using the relevant concepts, the bond graph of the quarter car model was utilized to derive the equations.

2.1 Bond Graph Model

For the purpose of simpler interpretation, only the suspension system above the tire was modeled.



The difference between an active and passive suspension is the presence of an actuator, which is represented by the controller in the diagram. Using this quarter car model, the following bond graph can be created



Using the bond graph, the state space equations can be derived. [8]

Passive control suspension:

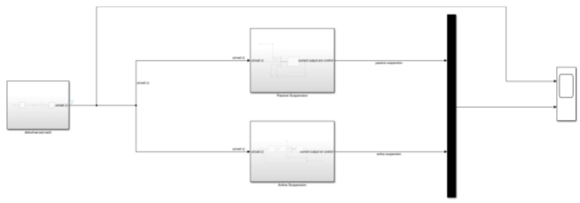
$$\begin{cases} \delta = v_{road} - \dfrac{p}{m} \\ \dot{p} = bv_{road} - mg - \dfrac{b}{m}p + k\delta \end{cases}$$

Active control suspension:

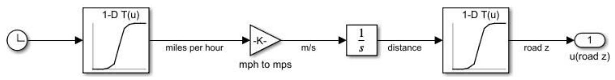
$$\begin{cases} \delta = v_{road} - \dfrac{p}{m} \\ \dot{p} = bv_{road} - mg - ctrl\ f - \dfrac{b}{m}p + k\delta \end{cases}$$

2.2 Simulink Model

The overall structure of the model is shown below and scope was used to check the results.

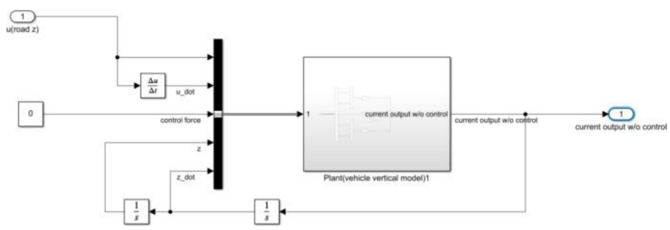


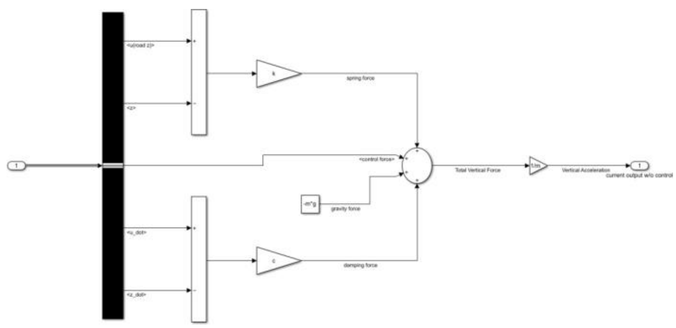
2.2.1 Input Section



spd_{prof} is a function that takes the different speeds of a car into consideration. Taking speed as a function of time, the unit is converted from miles per hour to m/s and is then integrated to calculate the distance travelled in real time. Using $road\ z$ and $road\ x$, the elevation of the road as a function of distance is converted such that itis a function of time and $u(road\ z)$ is outputted.

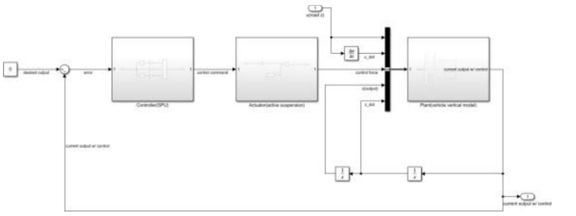
2.2.2 Passive Suspension



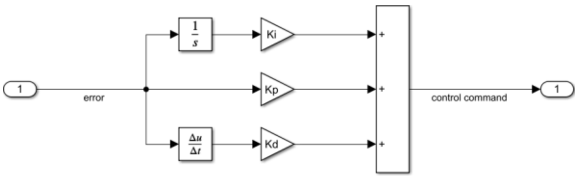


Using the bond graph from previously, the passive suspension model can be created using Simulink as shown above.

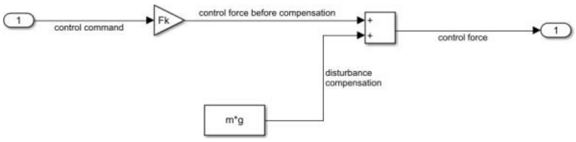
2.2.3 Active Suspension



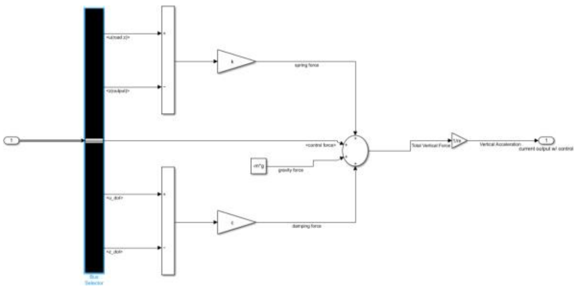
In terms of the bond graph for an active suspension, only the Se based on the actuator needs to be added. However, it is more complex to implement through Simulink, since a PID controller needs to be added. The overall configuration is shown below.



For the PID control, the proportional, integral, and derivative value of the error value is multiplied by the gain value and modeled.



Based on the inputted manipulated variable, the strength that results from the movement of the actuator and the weight of the car is taken into consideration to determine the control force. The Fk value was set arbitrarily for the sake of simplicity.

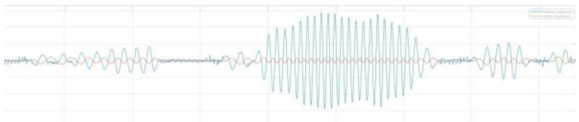


The model was created by considering the gravitational and control force of the plant, as well as the suspension's damper and spring force.

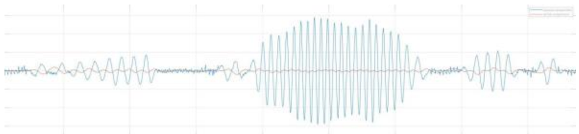
2.3 PID Gain Tuning

The initial value was found by using the Ziegler-Nichols method and after different adjustments of the value, theresult below was created. Since the acceleration in the Z-axis determines the ride quality, the goal is to minimizethe difference of this value through different tuning of the gain value.

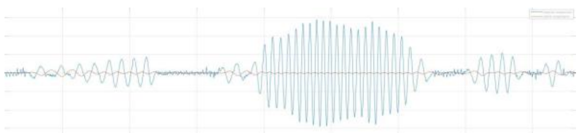
For the initial tuning, the graph below represents a value of 60 for K_p, 10 for K_i, and 12.5 for K_d.



For the intermediate tuning, the graph below represents a value of 70 for K_p, 10 for K_i, and 150 for K_d.



For the final tuning, the graph below represents a value of 60 for Kp, 3.6 for Ki, and 250 for Kd. As seen in the graph, there is minimal change in acceleration, and thus using such active suspension would increase the ride quality for the driver.



3 Conclusion

In conclusion, the quarter-car model and bond graph model were used to model the suspension system. PID control was utilized to find the gain value and improve the performance of the suspension system. In this investigation, only the suspension system above the tire was modeled to simplify the calculation and focus on a single goal. While modeling the tire as well would have been more complex, doing so could have yielded a more elaborate result with thorough analysis of the different variables.

Since PID control is heavily affected by even the smallest changes in the gain value, it is clear that tuning the gain value is essential. To find the suitable gain value, the Ziegler-Nichols method was first used to find the initial gain value. Not only did this method helped reduce the time taken when tuning using a trial and error process, but it also helped increase the performance.

The model used in this experiment was a quarter-car model, analyzing a model with only one wheel and movement in the *z*-axis. However, a real vehicle moves in all 6 axes, so there may be some difference between the model and a vehicle in real situations. Moreover, as the four wheels move in certain amounts, the road condition and the state of motion both affect the movement and should thus be analyzed for a more profound analysis.

Furthermore, the manipulated variable found through a simulation may not be the best value for a real vehicle. Therefore, it is important to check how a vehicle will behave in real situations and to replace the manipulated variable with a more optimal one if necessary. Even if the value may be the most optimal in a simulation, it may not be the most optimal in real life.

While this experiment only focused on controlling motion in the *z*-axis, the study of suspension control is extremely various. Therefore, for future studies, it would be insightful to explore the different suspension systems and design the most optimal control system for the best ride quality

Reference

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